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MATHEMATICAL STUDIES
STANDARD LEVEL
PAPER 1

Candidate session number

Tuesday 13 May 2014 (afternoon)

1 hour 30 minutes

Examination code

2	2	1	4	–	7	4	0	5

INSTRUCTIONS TO CANDIDATES

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the ***Mathematical Studies SL*** *formula booklet* is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is *[90 marks]*.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. The average radius of the orbit of the Earth around the Sun is 150 million kilometres.

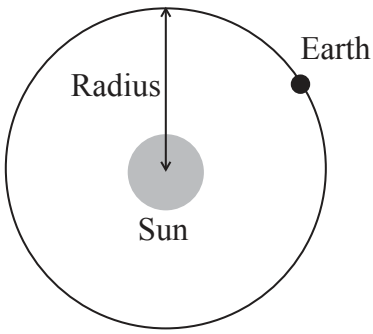


diagram not to scale

(a) Write down this radius, in kilometres, in the form $a \times 10^k$, where $1 \leq a < 10, k \in \mathbb{Z}$. *[2]*

The Earth travels around the Sun in one orbit. It takes one year for the Earth to complete one orbit.

(b) Calculate the distance, in kilometres, the Earth travels around the Sun in one orbit, assuming that the orbit is a circle. *[2]*

Today is Anna's 17th birthday.

(c) Calculate the total distance that Anna has travelled around the Sun, since she was born. *[2]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

2. Two propositions p and q are defined as follows

p: Eva is on a diet
q: Eva is losing weight.

(a) Write down the following statement **in words**.

$$q \Rightarrow p$$ *[2]*

(b) Write down, in words, the contrapositive statement of $q \Rightarrow p$. *[2]*

(c) Determine whether your statement in part (a) is logically equivalent to your statement in part (b). Justify your answer. *[2]*

Working:

Answers:

(a) . . . . . .

. . . . . .

. . . . . .

(b) . . . . . .

. . . . . .

(c) . . . . . .

. . . . . .

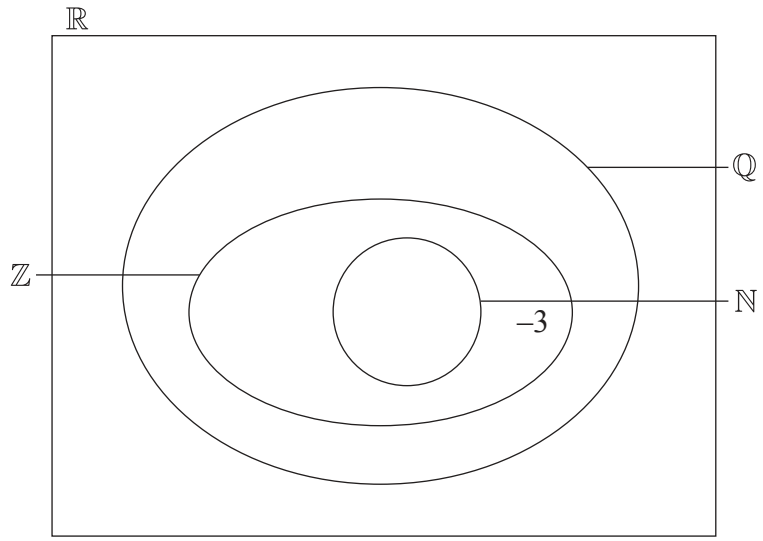
. . . . . .

. . . . . .

3. The following Venn diagram shows the relationship between the sets of numbers

$\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$.

The number –3 belongs to the set of $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$, but not $\mathbb{N}$, and is placed in the appropriate position on the Venn diagram as an example.



Write down the following numbers in the appropriate place in the Venn diagram.

(a) 4 *[1]*

(b) $\frac{1}{3}$ *[1]*

(c) π *[1]*

(d) 0.38 *[1]*

(e) $\sqrt{5}$ *[1]*

(f) –0.25 *[1]*

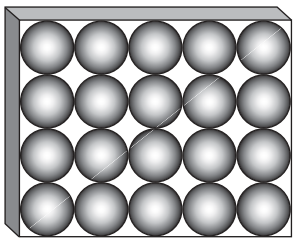
4. Chocolates in the shape of spheres are sold in boxes of 20.

Each chocolate has a radius of 1 cm.

(a) Find the volume of 1 chocolate. *[2]*

(b) Write down the volume of 20 chocolates. *[1]*

The diagram shows the chocolate box from above. The 20 chocolates fit perfectly in the box with each chocolate touching the ones around it or the sides of the box.



(c) Calculate the volume of the box. *[2]*

(d) Calculate the volume of empty space in the box. *[1]*

Working:

Answers:

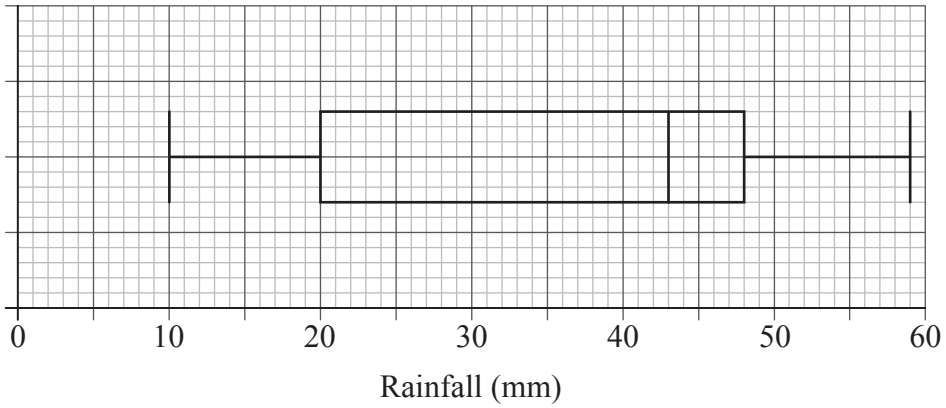
(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

5. The distribution of rainfall in a town over 80 days is displayed on the following box-and-whisker diagram.



(a) Write down the median rainfall. *[1]*

(b) Write down the minimum rainfall. *[1]*

(c) Find the interquartile range. *[2]*

(d) Write down the number of days the rainfall will be

(i) between 43 mm and 48 mm;

(ii) between 20 mm and 59 mm. *[2]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

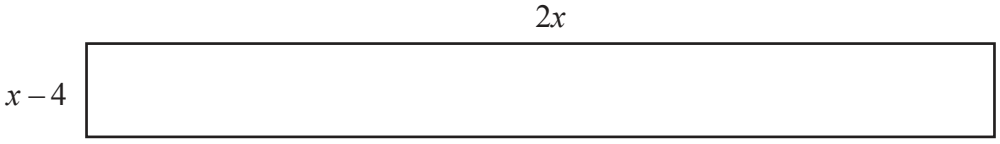
(c) . . . . . .

(d) (i) . . . . . .

(ii) . . . . . .

6. The surface of a red carpet is shown below. The dimensions of the carpet are in metres.

diagram not to scale



(a) Write down an expression for the area, A, in m^2, of the carpet. *[1]*

The area of the carpet is $10\ \text{m}^2$.

(b) Calculate the value of x. *[3]*

(c) Hence, write down the value of the length and of the width of the carpet, in metres. *[2]*

Working:

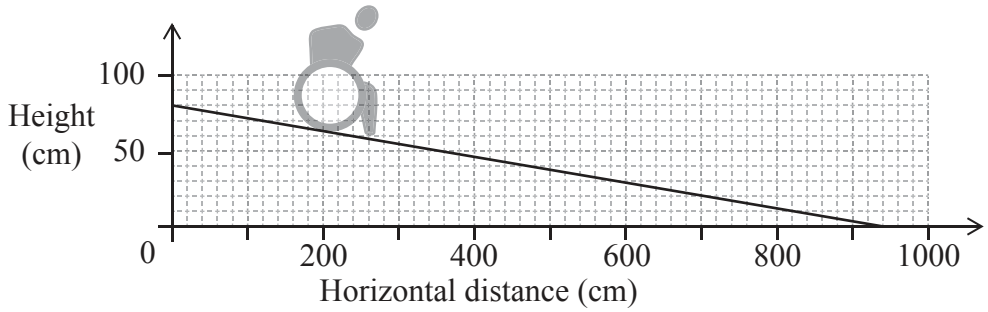
Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

7. The diagram shows a wheelchair ramp, A, designed to descend from a height of 80 cm.

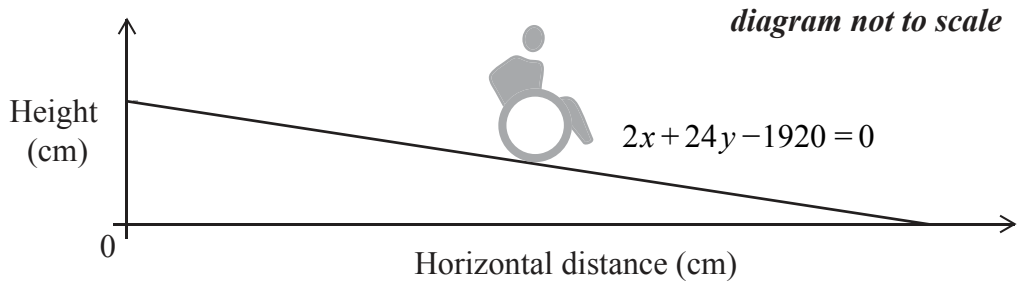


(a) Use the diagram above to calculate the gradient of the ramp. *[1]*

The gradient for a **safe** descending wheelchair ramp is $-\frac{1}{12}$.

(b) Using your answer to part (a), comment on why wheelchair ramp A is **not safe**. *[1]*

The equation of a second wheelchair ramp, B, is $2x + 24y - 1920 = 0$.



(c) (i) Determine whether wheelchair ramp B is safe or not. Justify your answer.

(ii) Find the horizontal distance of wheelchair ramp B. *[4]*

(This question continues on the following page)

(Question 7 continued)

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

. . . . . .

(c) (i) . . . . . .

. . . . . .

(ii) . . . . . .

8. A group of 100 students gave the following responses to the question of how they get to school.

	Walk	**Public transport**	**Car**	**Bicycle**	**Total**
Female	18	13	14	3	48
Male	9	17	10	16	52
Total	27	30	24	19	100

A χ^2 test for independence was conducted at the 5% significance level. The null hypothesis was defined as

H_0: Method of getting to school is independent of gender.

(a) Find the expected frequency for the females who use public transport to get to school. *[2]*

(b) Find the χ^2 statistic. *[2]*

The χ^2 critical value is 7.815 at the 5% significance level.

(c) State whether or not the null hypothesis is accepted. Give a reason for your answer. *[2]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

. . . . . .

. . . . . .

9. ***In this question give all answers correct to the nearest whole number.***

(a) Fumie is going for a holiday to Great Britain. She changes 100 000 Japanese Yen (JPY) into British Pounds (GBP) with no commission charged.

The exchange rate between GBP and JPY is

1 GBP = 129 JPY.

Calculate the value of 100 000 JPY in GBP. *[2]*

(b) At the end of Fumie's holiday in Great Britain she has 239 GBP. She converts this back to JPY at a bank, which does not charge commission, and receives 30 200 JPY.

(i) Find the exchange rate of this second transaction.

(ii) Determine, when changing GBP back to JPY, whether the exchange rate found in part (b)(i) is better value for Fumie than the exchange rate in part (a). Justify your answer. *[4]*

Working:

Answers:

(a) . . . . . .

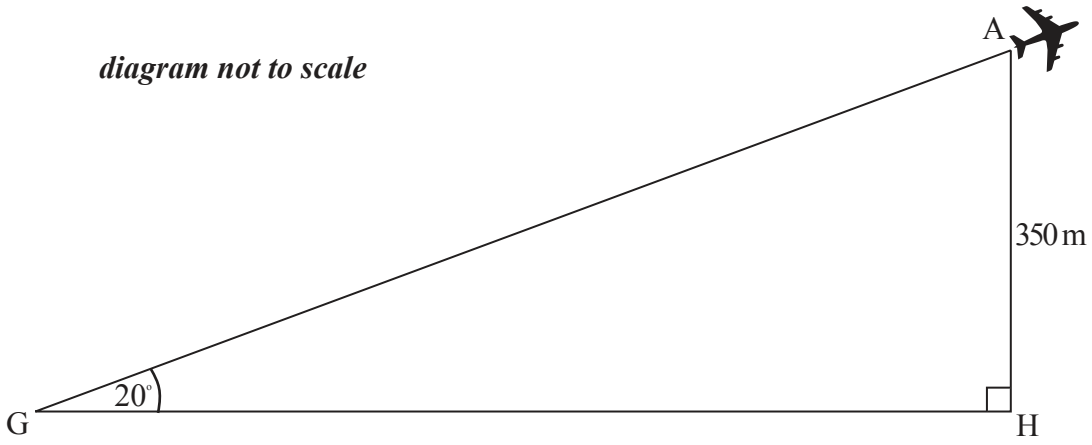
(b) (i) . . . . . .

(ii) . . . . . .

. . . . . .

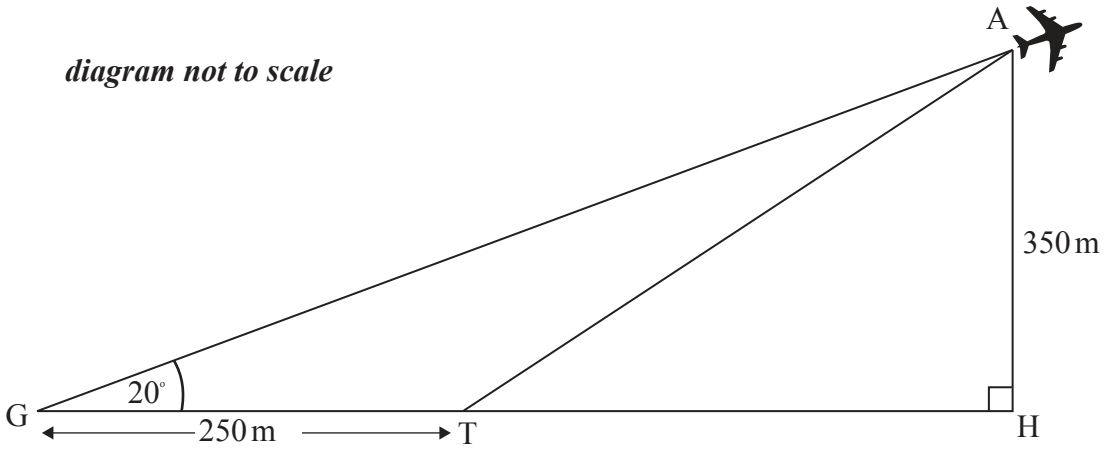
. . . . . .

10. Günter is at Berlin Tegel Airport watching the planes take off. He observes a plane that is at an angle of elevation of 20° from where he is standing at point G. The plane is at a height of 350 metres. This information is shown in the following diagram.



(a) Calculate the horizontal distance, GH, of the plane from Günter. **Give your answer to the nearest metre.** *[3]*

The plane took off from a point T, which is 250 metres from where Günter is standing, as shown in the following diagram.



(b) Using your answer from part (a), calculate the angle ATH, the takeoff angle of the plane. *[3]*

(This question continues on the following page)

(Question 10 continued)

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

11. In a trial for a new drug, scientists found that the amount of the drug in the bloodstream decreased over time, according to the model

$$D(t) = 1.2 \times (0.87)^t,\ t \geq 0$$

where D is the amount of the drug in the bloodstream in mg per litre $(\text{mg}\,\text{l}^{-1})$ and t is the time in hours.

(a) Write down the amount of the drug in the bloodstream at $t = 0$. *[1]*

(b) Calculate the amount of the drug in the bloodstream after 3 hours. *[2]*

(c) Use your graphic display calculator to determine the time it takes for the amount of the drug in the bloodstream to decrease to $0.333\,\text{mg}\,\text{l}^{-1}$. *[3]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

12. A survey investigated the relationship between the number of cleaners, n, and the amount of time, t, it takes them to clean a school.

Number of cleaners, n	**Time, t (minutes)**
1	193
2	172
3	118
5	112
6	87

(a) Use your graphic display calculator to write down the equation of the regression line t on n. *[2]*

(b) Write down the value of the Pearson's product–moment correlation coefficient, r. *[2]*

(c) Use your regression equation to find the amount of time 4 cleaners take to clean the school. *[2]*

Working:

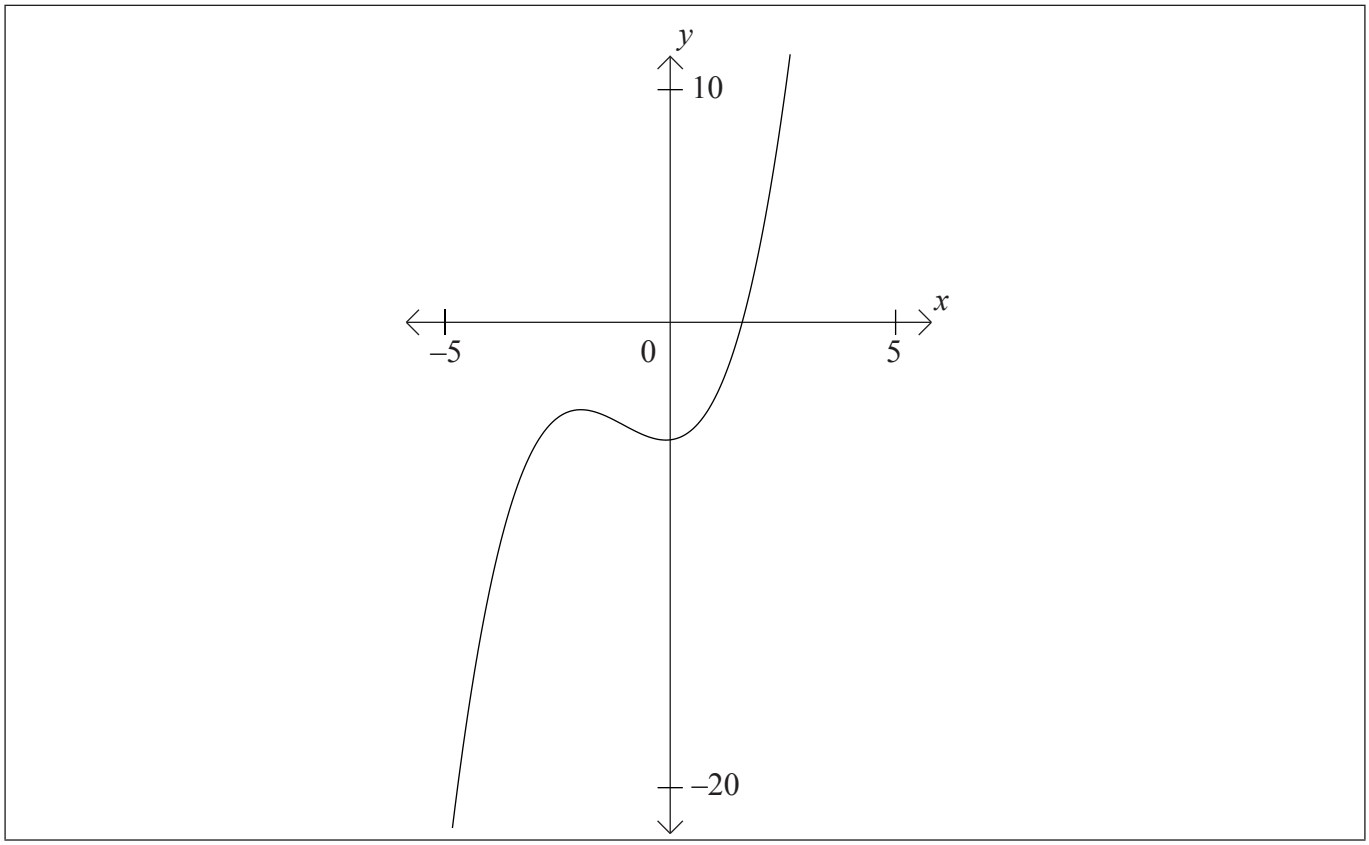
Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

13. Consider the graph of the function $f(x) = x^3 + 2x^2 - 5$.



(a) Label the local maximum as A on the graph. *[1]*

(b) Label the local minimum as B on the graph. *[1]*

(c) Write down the interval where $f'(x) < 0$. *[1]*

(d) Draw the tangent to the curve at $x = 1$ on the graph. *[1]*

(e) Write down the equation of the tangent at $x = 1$. *[2]*

(This question continues on the following page)

(Question 13 continued)

Working:

Answers:

(c) . . . . . .

(e) . . . . . .

14. The heights of apple trees in an orchard are normally distributed with a mean of 3.42 m and a standard deviation of 0.21 m.

(a) Write down the probability that a randomly chosen tree has a height greater than 3.42 m. *[1]*

(b) Write down the probability that a randomly chosen tree will be within 2 standard deviations of the mean of 3.42 m. *[1]*

(c) Use your graphic display calculator to calculate the probability that a randomly chosen tree will have a height greater than 3.35 m. *[2]*

(d) The probability that a particular tree is less than x metres high is 0.65. Find the value of x. *[2]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

(c) . . . . . .

(d) . . . . . .

15. A function is given as $f(x) = 2x^3 - 5x + \frac{4}{x} + 3,\ -5 \le x \le 10,\ x \neq 0$.

(a) Write down the derivative of the function. *[4]*

(b) Use your graphic display calculator to find the coordinates of the local minimum point of $f(x)$ in the given domain. *[2]*

Working:

Answers:

(a) . . . . . .

(b) . . . . . .

Please **do not** write on this page.

Answers written on this page
will not be marked.